Name:_____ Date:_____

Precalculus 11: HW 1.1 Arithmetic Sequences Solutions

1. Given each of the following sequences, indicate whether if it's an arithmetic sequence. Explain why or why not:

a) $2,\ 8,\ 14,\ 20,\ldots$ Yes, each successive term increases by 6	b) $4,\ 7,\ 11,\ 16,\ 22,\ldots$ No because each term increases by a different value
c) $3,\ 9,\ 27,\ 81\ldots$ No because each term is multiplied by 3	d) $2,\ 4,\ 8,\ 16,\ 32\ldots$ No because each term is multiplied by 2
e) $100, 90, 80, 70, 60, \ldots$ Yes because each term is decreased by the same value of 10	f) $\frac{4}{3},\ \frac{3}{2},\ \frac{5}{3},\ \frac{11}{6}\ldots$ Yes because each term is increased by 1/6

2. Given each arithmetic sequence below, find the value of the first term t_1, common difference d, and the value of the n^{th} term t_n that is requested: Use the equation $\boxed{t_n = a + (n-1)\times d}$ to evaluate t_n :

	t_1 or a	d	$t_7\ (7^{th}\ term)$	$t_{11}\ (11^{th}\ term)$
i) $41,\ 34,\ 27,\ 20\ldots$	41	-7	$t_7 = 41+(7-1)(-7)$ $t_7 = 41+(-42)$ $t_7 = -1$	$t_{11} = 41+(11-1)(-7)$ $t_{11} = 41+(-70)$ $t_{11} = -39$
ii) $4,\ 10,\ 16,\ 22,\ \ldots$	4	6	$t_7 = 4+(7-1)(6)$ $t_7 = 4+(36)$ $t_7 = 40$	$t_{11} = 4+(11-1)(6)$ $t_{11} = 4+(60)$ $t_{11} = 64$
iii) $-24, -12,\ 0,\ 12,\ \ldots$	-24	-12	$t_7 = -24+(7-1)(-12)$ $t_7 = -24+(-72)$ $t_7 = -96$	$t_{11} = -24+(11-1)(-12)$ $t_{11} = -24+(-120)$ $t_{11} = -144$
iv) $\frac{24}{5},\ \frac{14}{5},\ \frac{4}{5},\ \frac{-6}{5},\ \ldots$	$\frac{24}{5}$	-2	$t_7 = \frac{24}{5}+(7-1)(-2)$ $t_7 = \frac{24}{5}+(-12)$ $t_7 = -7.2$	$t_{11} = \frac{24}{5}+(11-1)(-2)$ $t_{11} = \frac{24}{5}+(-20)$ $t_{11} = -15.2$
v) $\frac{22}{3},\ \frac{47}{6},\ \frac{25}{3},\ \frac{53}{6},\ \ldots$	$\frac{22}{3}$	$\frac{1}{2}$	$t_7 = \frac{22}{3}+(7-1)(-0.5)$ $t_7 = \frac{22}{3}+(-3)$ $t_7 = \frac{13}{3}$	$t_{11} = \frac{22}{3}+(11-1)(-0.5)$ $t_{11} = \frac{22}{3}+(-5)$ $t_{11} = \frac{7}{3}$

3. Find the missing values for each of the arithmetic sequences. Show all your work:

a) $\square,\ \square,\ \square,\ 17,\ 21$

$t_5 - t_4 = d$
$21-17 = 4$
$d = 4$

$t_3 = 17-4$ $t_2 = 13-4$ $t_1 = 9-4$
$t_3 = 13$ $t_2 = 9$ $t_1 = 5$

b) $\square,\ \square,\ 26,\ \square,\ 16$

$t_5 - t_3 = 2d$
$16-26 = -10$ $t_4 = 16-(-5)$ $t_2 = 26-(-5)$ $t_1 = 31-(-5)$
$2d = -10$ $t_4 = 21$ $t_2 = 31$ $t_1 = 36$
$d = -5$

c) $108,\ \square,\ \square,\ \square,\ 40$

$t_5 - t_1 = 4d$
$40-108 = -68$ $t_4 = 40-(-17)$ $t_3 = 57-(-17)$ $t_2 = 74-(-17)$
$4d = -68$ $t_4 = 57$ $t_3 = 74$ $t_2 = 91$
$d = -17$

d) $\square,\ \square,\ 14,\ 18,\ 21$

$t_5 - t_4 = d$ $t_4 - t_3 = d$
$21-18 = 3$ $18-14 = 4$
$d = 3$ $d = 4$

the value of "d" is inconsistent, therefore it is not an arithmetic sequence

4. Given the equation, find the value of the first 3 terms:

a) $t_n = 4 + 5n$

$t_1 = 4 + 5(1)$ $t_2 = 4 + 5(2)$ $t_3 = 4 + 5(3)$

$t_1 = 9$ $t_2 = 14$ $t_3 = 19$

b) $t_n = 7 + (n-1)8$

$t_1 = 7 + (1-1)8$ $t_2 = 7 + (2-1)8$ $t_3 = 7 + (3-1)8$

$t_1 = 7$ $t_2 = 15$ $t_3 = 23$

5. Given the following arithmetic sequences, find out how many terms there are and the equation of the General Term: Use the equation $\boxed{t_n = a + (n-1) \times d}$ to evaluate "n" : Show all your work:

a) $5,\ 9,\ 13,\ldots..\ 209$

$a = 5, d = 4, t_n = 209$ $209 = 5 + 4n - 4$

$t_n = a + (n-1)d$ $209 = 1 + 4n$ $52 = n$

$209 = 5 + (n-1)4$ $208 = 4n$

Number of terms 52 General Term:

$t_n = 5 + (n-1)4$

b) $-210,\ -207.5,\ -205,\ldots...45$

$a = -210, d = 2.5, t_n = 45$ $45 = -210 + 2.5n - 2.5$

$t_n = a + (n-1)d$ $45 = -212.5 + 2.5n$ $103 = n$

$45 = -210 + (n-1)2.5$ $257.5 = 2.5n$

Number of terms 103 General Term:

$t_n = -210 + (n-1)2.5$

c) $\frac{1}{2},\ \frac{7}{6},\ \frac{11}{6},\ldots..\ \frac{29}{2}$

$a = 0.5, d = \frac{2}{3}, t_n = \frac{29}{2}$ $14.5 = 0.5 + \frac{2}{3}n - \frac{2}{3}$

$t_n = a + (n-1)d$ $14.5 = -\frac{1}{6} + \frac{2}{3}n$ $22 = n$

$14.5 = 0.5 + (n-1)\frac{2}{3}$ $\frac{44}{3} = \frac{2}{3}n$

Number of terms 22 General Term:

$t_n = \frac{1}{2} + (n-1)\frac{2}{3}$

d) $\frac{4}{5},\ \frac{2}{15},\ \frac{-8}{15},\ldots..\ \frac{-126}{5}$

$a = 0.8, d = \frac{2}{3}, t_n = \frac{-126}{5}$ $\frac{-126}{5} = 0.8 + \frac{2}{3}n - \frac{2}{3}$ $\frac{76}{3} = \frac{2}{3}n$

$t_n = a + (n-1)d$ $\frac{-126}{5} = \frac{2}{15} + \frac{2}{3}n$ $38 = n$

$\frac{-126}{5} = 0.8 + (n-1)\frac{2}{3}$ $\frac{380}{15} = \frac{2}{3}n$

Number of terms 38 General Term:

$t_n = \frac{4}{5} + (n-1)\frac{2}{3}$

6. Jimmy works as a vendor at the PNE selling frosted malts. His boss pays him \$47 on the first day of work and increases his pay by \$2.50 a day after each month. How much will he earn a day after working 13 months?

Let the first term by \$47 and the common difference be \$2.50, and "n" be 13

$t_{13} = 47 + (13-1)2.5$

$t_{13} = 47 + (12)2.5$

$t_{13} = \$77$

Jimmy would earn \$77 per day

7. A sequence $a_1, a_2, a_3, a_4, \ldots..$ is an arithmetic sequence if and only if:

a) All the terms are positive

b) All the terms are getting bigger

c) All the terms have a positive difference

d) All the terms have the same difference

The answer is "d", all successive terms must have the same difference to be an arithmetic sequence

8. What is the seventh term of an arithmetic sequence with a first term of nine and a common difference of twelve?

$t_7 = 9 + (7-1)12$

$t_7 = 9 + (6)12$

$t_7 = 81$

9. What is the common difference of an arithmetic sequence if the second term is 4 and the eight terms is 100?

$t_8 - t_2 = 6d$

$100 - 4 = 96$

$6d = 96$

$d = 16$

10. How many terms are in a sequence if the first term is 14, fourth term is 38, and the last term is 190?

$t_4 - t_1 = 3d$

$38 - 14 = 24$

$3d = 24$

$d = 8$

$t_n = a + (n-1)d$

$190 = 14 + (n-1)8$

$190 = 14 + 8n - 8$

$190 = 6 + 8n$

$184 = 8n$

$23 = n$

There are 23 terms

11. The first three terms of an arithmetic sequence is given by the following expressions: $2x+1,\ 4x,\ 5x+2$. Find the value of "x" and the value of each term:

$t_1 = 2x+1$

$t_2 = 4x$

$t_3 = 5x+2$

$t_2 - t_1 = t_3 - t_2$

$4x - (2x+1) = (5x+2) - 4x$

$4x - 2x - 1 = 5x + 2 - 4x$

$2x - 1 = x + 2$

$x = 3$

$t_1 = 2(3)+1 = 7$

$t_2 = 4(3) = 12$

$t_3 = 5(3)+2 = 17$

12. Consider the following arithmetic sequence, which term is the first positive term? $-16,\ -14.75,\ -13.5...$

You can either make a list by adding 1.25 to each term or use the formula:

$t_n = -16 + (n-1)(1.25)$

$t_n = -16 + (n-1)(1.25)$

If "n" is 13, then:

$t_{13} = -16 + (13-1)(1.25)$

$t_{13} = -16 + (15)$

$t_{13} = -1$

now try n=14

$t_{14} = -16 + (14-1)(1.25)$

$t_{14} = -16 + (16.25)$

$t_{14} = 0.25$

Therefore the 14th term is positive

13. A nine term arithmetic sequence $a_1, a_2, a_3, a_4, \ldots\ldots$ satisfies $a_1 + a_3 = 14$ and $a_2 + a_4 = 25$. What is the value of the first four terms?

Express a2, a3, and a4 in terms of a1

$$a_2 = a_1 + d$$
$$a_3 = a_1 + 2d$$
$$a_4 = a_1 + 3d$$

Rewrite the two equations:

$$a_1 + a_1 + 2d = 14$$
$$a_1 + d + a_1 + 3d = 25$$

Simplify both equations

$$\rightarrow \quad 2a_1 + 2d = 14 \quad \rightarrow \quad a_1 + d = 7$$
$$\rightarrow \quad 2a_1 + 4d = 25 \quad \rightarrow \quad a_1 + 2d = 12.5$$

If you subtract the two equations, you will get the value of "d"

$$\begin{array}{l} a_1 + d = 7 \\ \underline{a_1 + 2d = 12.5} \\ d = 5.5 \end{array}$$

If d=5.5, solve for the first term

$$a_1 + (5.5) = 7 \qquad a_2 = 1.5 + 5.5 \qquad a_3 = 7 + 5.5 \qquad a_4 = 12.5 + 5.5$$
$$a_1 = 1.5 \qquad a_2 = 7 \qquad a_3 = 12.5 \qquad a_4 = 18$$

Now check your answers:

$a_1 + a_3 = 14$ $\qquad$ $a_2 + a_4 = 25$

$1.5 + 12.5 = 14$ $\qquad$ $7 + 18 = 25$

correct! $\qquad$ *correct*